Chapter 13: Breakeven Analysis

Breakeven analysis is performed to determine the value of a variable of a project that makes two elements equal, e.g. sales volume that will equate revenues and costs.

Single Project

The analysis is based on the relationship:

Profit = revenue – total cost
= R – TC

At breakeven, there is no profit or loss, hence,
revenue = total cost

or,   R = TC

Note: It is to be noted that +ve sign is used for both the revenue and the costs. If we are to use –ve sign for costs and +ve sign for revenue, then the above relationships become:

Profit = R + TC   and   R + TC = 0   at breakeven.

With revenue and costs given in terms of a decision variable, the solution yields the breakeven quantity for the decision variable.

**Costs**, which may be linear or non-linear, usually include two components:

Fixed costs (FC) – Includes costs such as buildings, insurance, fixed overhead, equipment capital recovery, etc. These costs are essentially constant for all values of the decision variable.

Variable costs (VC) – Includes costs such as direct labour, materials, contractors, marketing, advertisement, etc. These costs change linearly or non-linearly with the decision variable, e.g. production level, workforce size, etc. For the analysis to be followed here, the variation will generally be assumed to be linear.

Then, total cost,   TC = FC + VC

Revenue also changes with the decision variable. Again, for the analysis, the variation will generally be assumed to be linear.

The following diagram illustrates the basics of the breakeven analysis.

Production, Q units/year
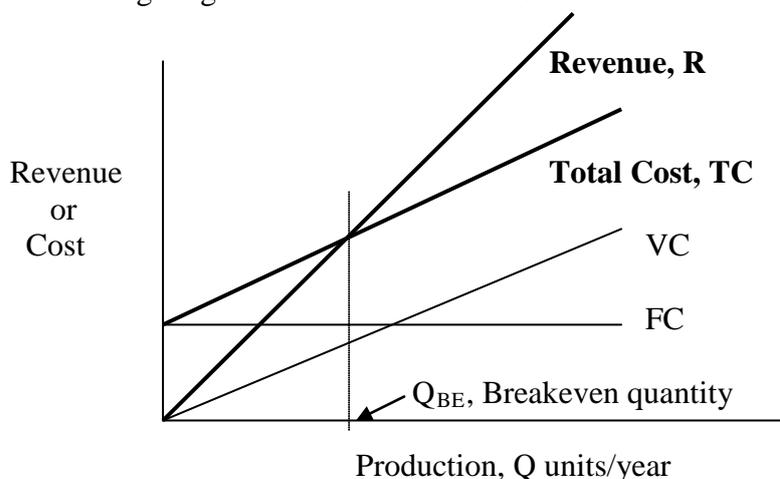
It can be seen that we have profit if the production level is above the breakeven quantity and loss if it is below.

Examples:

1. The fixed costs at Company X are $1 million annually. The main product has revenue of $8.90 per unit and $4.50 variable cost. (a) Determine the breakeven quantity per year, and (b) Annual profit if 200000 units are sold.

Let Q_{BE} be the breakeven quantity.

$$8.9Q_{BE} = 1,000,000 + 4.5Q_{BE}$$

$$Q_{BE} = 1,000,000/(8.90\text{-}4.50) = 227,272 \text{ units}$$

(b)   Profit = R – TC

$$= 8.90Q - 1,000,000 - 4.5Q$$

At 200,000 units: Profit = 8.90(200,000) – 1,000,000 - 4.50(200,000)
                         = $-120,000 (loss)

2. A product currently sells for $12 per unit. The variable costs are $4 per unit, and 10,000 units are sold annually and a profit of $30,000 is realized per year. A new design will increase the variable costs by %20 and Fixed Costs by %10 but sales will increase to 12,000 units per year. (a) At what selling price do we break even, and (b) If the selling price is to be kept same ($12/unit) what will the annual profit be?

Profit = revenue – costs

30000 = 10000(12) – [10000(4) + FC]   FC = fixed costs

FC = 50000

(a)   New variable cost = $4(1.2) = $4.8 per unit.

New fixed costs = 50000(1.1) = $55000

Let x = breakeven selling price per unit, then

12000x = 55000 + 12000(4.8)
or,   x = $9.38/unit

(b)   Profit = 12000(12) – 12000(4.8) - 55000
            = $31400

3. A defense contractor has been able to summarize its total annual fixed costs as $100,000 and the total variable cost per unit of production as $33. (a) If only 5000 units is all that is expected to sell to the government this year what should the per unit selling price be to make a %25 profit this year? (b) If foreign sales of 3000 units per year is to be added to the 5000 units government contract above and a %25 profit is acceptable for this contractor again, what could be the new selling price per unit?

a)   Total costs = 100000 + 5000(33)
                = 265000

     % profit = 100(revenue – cost)/cost

Therefore,   25 = 100(revenue – 265000)/265000

or,   revenue = 265000(1.25) = 331250

     Selling price = 331250/5000
                = $66.25 / unit.

b)   Total cost = 100000 + 8000(33) = 364000

     Revenue for 25% profit = 364000(1.25) = 455000

     New selling price = 455000/8000
                = $56.875 per unit.

4. Suppose a firm is considering manufacturing a new product and the following data have been provided:

Sales price	$12.50 per unit
Equipment cost	$200 000
Overhead cost	$50 000 per year
Operating and maintenance cost	$25 per operating hour
Production time	0.1 hours per unit
Planning period	5 years
MARR	15%

Assuming a zero salvage value for all equipment at the end of five years, determine the number of unit to be produced to break even.

Let X = number of units to be provided per year to break even.
     $AW_C = -200000.(A/P,15\%,5) – 50000 – (0.1)25X$
          = -109660 – 2.5X

     Revenue: $AW_R = 12.5X$

At breakeven,   12.5X – 109660 – 2.5X = 0

          X = 10966 units per year.

Note: -ve sign for costs and +ve sign for revenue is used in the above solution.

5. An automobile company is planning to convert a plant from manufacturing economy cars to manufacturing sports cars. The initial cost for equipment conversion will be $200 million with a 20% salvage value anytime within a 5-year period. The cost of producing a car will be $21000, and it will be sold for $33000. The production capacity for the first year will be 4000 units. At an interest rate of 12% per year, by what uniform amount will production have to increase each year in order for the company to recover its investment in 3 years?

Let x = gradient increase per year.

Total costs = -200M(A/P,12%,3)+(0.20)(200M)(A/F,12%,3)-[4000+ x(A/G,12%,3)](21,000)

Revenue = [4000 + x(A/G,12%,3)](33,000)

At breakeven, revenue + costs* = 0, then   * -ve sign for costs is used.

[4000 + x(A/G,12%,3)](33,000 – 21,000) = 200M(A/P,12%,3) - (0.20)(200M)(A/F,12%,3)

[4000 + x(0.9246)](12,000) = 200M(0.41635) - 40M(0.29635)

    x = 2110 cars/year increase

6. Owners of a hotel chain are considering locating a new hotel in Karpaz. The complete cost of building a 150-room hotel (excluding furnishings) is $2million; the furnishings will cost $750 000 and will be replaced every 5 years for the same cost. Annual operating and maintenance cost for the facility is estimated to be $50 000. The average rate for a room is expected to be $15 per day. A 15-year planning period is used by the firm in evaluating new projects of this type; a terminal salvage value of 20% of the original building cost is anticipated; furnishings are estimated to have no salvage value at the end of each five-year replacement interval; land cost is not to be included. Determine the break-even value for the average number of rooms to be occupied daily based on a MARR of 10% (Assume the hotel will operate 365 days a year).

Annualizing Costs:

Building: AW_B = -2M(A/P,10%,15) = -2M(0.13147) = -262940 / yr
Furnishings: AW_F = [-750000 – 750000(P/F,10%,5) – 750000(P/F,10%,10)](A/P,10%,15)

            = -197836.06 / yr

Salvage Value = (0.2)2M = 400000
        AW_S = 400000(A/F,10%,15) = 400000(0.03147) = 12588

Total Annual Cost, AW_C = -262940 –197836 – 50000 + 12588 = -498188

Revenue = 15(365)X, where X is number of rooms occupied.

At break-even,

        15(365)X – 498188 = 0

        or X = 91 rooms per day on the average.

Two or more Alternatives

This is commonly applied to between alternatives that serve the same purpose. As a result, breakeven analysis is carried out between the costs of the alternatives. It involves the determination of a common variable between two or more alternatives. The procedure to follow for two alternatives is as follows:

- Define the common variable and its dimensional units.
- Use AW or PW analysis to express the total cost of each alternative as a function of the common variable. (Use AW values if lives are different).
- Equate the two relations and solve for the breakeven value of the variable.
- If the anticipated level is below the breakeven value, select the alternative with the higher variable cost (larger slope). If the level is above the breakeven point, select the alternative with the lower variable cost.

The same type of analysis can be performed for three or more alternatives. Then, compare the alternatives in pairs to find their respective breakeven points. The results are the ranges through which each alternative is more economical.

Examples:

7. A Textile company is evaluating the purchase of an automatic cloth-cutting machine. The machine will have a first cost of $22000, a life of 10 years, and a $500 salvage value. The annual maintenance cost of the machine is expected to be $2000 per year. The machine will require one operator at a total cost of $24 an hour. Approximately 1500 meters of material can be cut each hour with the machine.
Alternatively, if human labor is used, five workers , each earning $10 an hour , can cut 1000 meters per hour. If the company's MARR is %8 per year , and 180,000 meters of material is to be cut every year should the company buy the automatic machine or use human labor instead?
At how many meters cloth-cutting per year will the two alternatives breakeven?

Let x = meters of material to be cut

Automatic machine:
Total annual cost, AW_A = -22000(A/P,8%,10) + 500(A/F,8%,10) – 2000 – (x/1500)(24)

$$= -5244.15 – x/62.5$$

Manual:
Total annual cost, AW_M = -(x/1000)(5)(10) = -x/20

At breakeven, $AW_A = AW_M$

$$-5244.15 – x/62.5 = x/20$$

or,   x = 154240 m

Therefore, at 180000 m, select the automatic machine. (we make profit if quantity is above the breakeven).

8. Two types of pumps are available. Pump X costs $800 and has a life of 3 years. It also requires rebuilding after 2000 operating hours at a cost of $300. Pump Y costs $1900 and is expected to last 5 years. It also requires overhaul after 8000 hours of operation at a cost of $700. If the operating cost of each pump is $1 per hour, how many hours per year must the pump be required to justify the purchase of pump Y? (Interest rate = 10% per year).

Let x = hours per year

Since lives are different, it is best to use the AW values.

Annual cost of X: $AW_X = -800(A/P,10\%,3) - (300/2000)x - 1.0x$

Annual cost of Y: $AW_Y = -1,900(A/P,10\%,5) - (700/8000)x - 1.0x$

At breakeven, $AW_X = AW_Y,$

$-800(A/P,10\%,3) - (300/2000)x - 1.0x = -1,900(A/P,10\%,5) - (700/8000)x - 1.0x$
$-800(0.40211) - 0.15x - 1.0x = -1,900(0.2638) - 0.0875x - 1.0x$
$0.0625x = 179.532$

        $x = 2873$ hours per year

9. Machine A has a fixed cost of $40000 per year and a variable cost of $60 per unit. Machine B has an unknown fixed cost, but with this process 200 units can be produced each month at a total variable cost of $2000. If the total costs of the two machines break even at a production rate of 2000 units per year, what is the fixed cost of machine B?

Let FCB = fixed cost for B.

Variable cost for B = 2000/200 = $10/unit

Total costs are equal at 2000 units per year. Then,

$40,000 + 60(2000 \text{ units}) = FCB + 10(2000 \text{ units})$

FCB = $140,000 per year

10. The ABC Company is faced with three proposed methods for making one of their products. Method A involves the purchase of a machine for $5000. It will have a seven-year life, with a zero salvage value at that time. Using Method A involves additional costs of $0.20 per unit of product produced per year. Method B involves the purchase of a machine for $10000. It will also have a seven-year life, with $2000 salvage value at that time. Using Method B involves additional costs of $0.15 per unit of product produced per year. Method C involves the purchase of a machine for $8000. It will have a $2000 salvage value when disposed of in seven years. Additional costs of $0.25 per unit of product per year arise when Method C is used. An 8% interest rate is used by the ABC Company in evaluating investment alternatives. For what range of annual production volume values is each method preferred?

Let X = number of units per year

$$AW_A = -5000 \cdot (A/P,8\%,7) - 0.2X = -5000 \cdot (0.19207) - 0.2X$$
$$= -960.35 - 0.2X$$

$$AW_B = -10000 \cdot (A/P,8\%,7) + 2000 \cdot (A/F,8\%,7) - 0.15X = -10000 \cdot (0.19207) +$$
$$2000 \cdot (0.11207) - 0.15X$$
$$= -1696.56 - 0.15X$$

$$AW_C = -8000 \cdot (A/P,8\%,7) + 2000 \cdot (A/F,8\%,7) - 0.25X$$
$$= -1312.42 - 0.25X$$

A vs B: $-960.35 - 0.2X = -1696.56 - 0.15X$
$$X_{AB} = 14724.2$$

B vs C: $-1696.56 - 0.15X = -1312.42 - 0.25X$
$$X_{BC} = 3841.4$$

A vs C: No intersection for $X > 0$.

X ≤ 14724, select A
X > 14724, select B
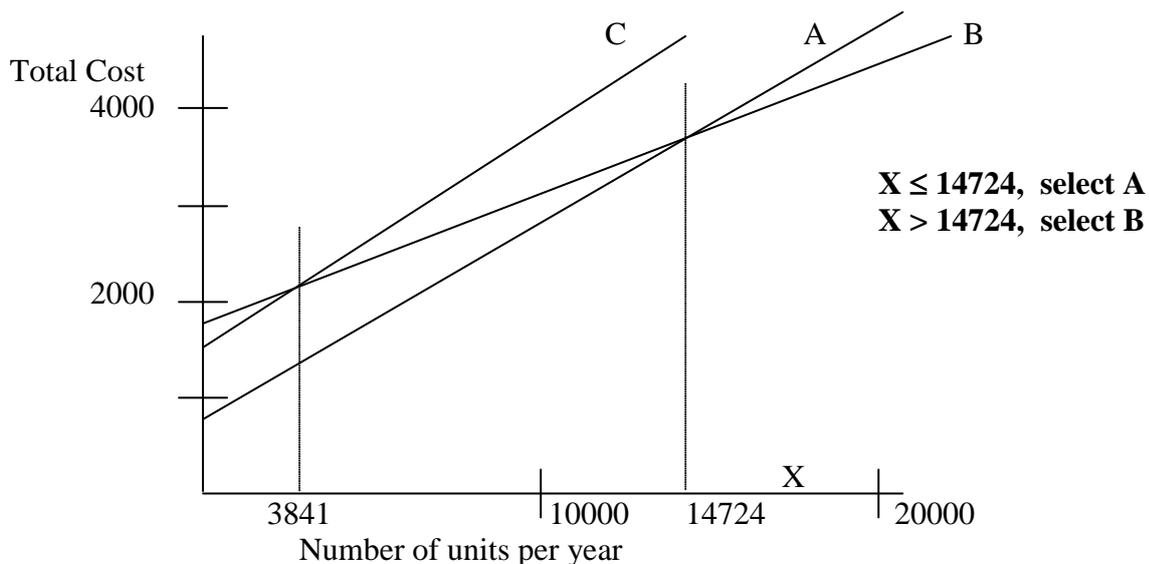
11. Three types of design proposals for a commercial one - storey building is to be evaluated details given below:

	STEEL	CONCRETE	BRICK
First cost	$72/ft^2	$76/ft^2	$81/ft^2
Annual maintenance	$14000	$9000	$6000
Annual heating cost	$3/ft^2	$3.4/ft^2	$3.9/ft^2
SV (%of first cost)	%80	%100	%110
Life (years)	20	20	20

For what range of building area (ft^2) which type of design is the most suitable (cheapest) to select? Carry out breakeven analysis using an interest rate of %18 per year and plot your ranges to illustrate.

Let X = area in ft^2

First cost of steel = 72x   and its salvage value = (0.8)72x

Using AW values:

AW_S = -(72x)(A/P,18%,20) + (0.8)(72x) – 14000 – 3x
       = -13.45x + 0.393x – 14000 – 3x
       = -16.057x – 14000

Similarly,

AW_C = -(76x)(A/P,18%,20) + (1.0)(76x) – 9000 – 3.4x
       = -17.082x – 9000

AW_B = -(81x)(A/P,18%,20) + (1.1)(81x) – 6000 – 3.9x
       = -18.423x – 6000

Breakevens:

Steel vs Concrete:
          -16.057x – 14000 = -17.082x – 9000
                    x = 4878 ft^2

Steel vs Brick:
          -16.057x – 14000 = -18.423x – 6000
                    x = 3381 ft^2

Concrete vs Brick:
          -17.082x – 9000 = -18.423x – 6000
                    x = 2237 ft^2
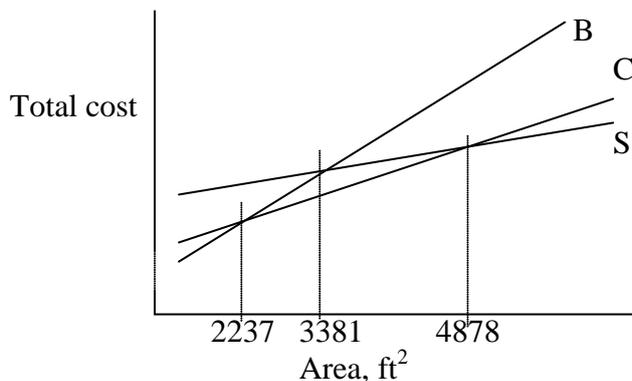
$0 < x \le 2237$ Select Brick
$2237 < x \le 4878$ Select Concrete
$x > 4878$ Select Steel

12. Three options are considered for an engine part:

A – complete in-house manufacturing, with initial equipment cost of $50 000, labor cost of $26 000 per year, and material cost of $10 per engine part.
B – partial manufacture, (i.e. partially finished engine parts are purchased), with initial equipment cost of $35 000, labor cost of $10 000 per year, material cost of $3 per engine part, and an additional cost of $40 per the partially finished engine part.
C – purchase from outside at a cost of $120 per engine part.
Any equipment purchased will have a life of 6 years. If the MARR is 10% per year, determine the number of engine parts that must be manufactured to justify (a) complete in-house manufacture and (b) partial manufacture. (c) Plot the total cost lines for all three options, and state the ranges of engine parts for which each option will have the lowest cost.

Let x = number of engine parts per year.

Using AW values:

$$AW_{IN} = -50000(A/P,10\%,6) - 26000 - 10x$$

$$AW_{PM} = -35000(A/P,10\%,6) - 10000 - 3x - 40x$$

$$AW_{OUT} = -120x$$

(a) Complete in-house manufacturing vs purchase from outside:
$$AW_{IN} = AW_{OUT}$$

or, $-50,000(A/P,10\%,6) - 26,000 - 10x = -120x$

x = 341 parts per year

(b) Partial manufacture vs purchase from outside:
$$AW_{PM} = AW_{OUT}$$

$-35,000(A/P,10\%,6) - 10,000 - 3x - 40x = -120x$

x = 234 parts per year

(c) Complete in-house vs partial manufacture
$$AW_{IN} = AW_{PM}$$

$-50,000(A/P,10\%,6) - 26,000 - 10x = -35,000(A/P,10\%,6) - 10,000 - 3x - 40x$

x = 589 parts per year

Ranges for the lowest total cost are:

$0 < x \leq \square 234$ select purchase fro outside

$234 < x \leq \square 589$ select partial manufacture

$589 < x$ select in-house manufacture