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Lecture notes-10
BCA I year
Topic- Lattices

VARDHMAN
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Theorem-2
P-117

Let $(L, \leq)$ be a lattice in which $\wedge$ and $\vee$ denote the operation meet and joint. $a, b \in L$ Then show that

(a) $a \leq b \iff a \wedge b = a$
(b) $a \leq b \iff a \vee b = b$

Proof (a) Let $a \wedge b = a$ Then show $a \leq b$

i.e

$$a \wedge b = \inf\{a, b\}$$
$$\Rightarrow a = \inf\{a, b\} \quad \{\because a \wedge b = a$$
$$\Rightarrow a \leq a, \; a \leq b$$
$$\Rightarrow a \leq b$$

Converse $\rightarrow$

Let $a \leq b$ Then show $a \wedge b = a$

i.e a is Partially order related to b

So It is reflexive Then $a \leq a$

But $a \leq b$

(2) $\Rightarrow a \leq a, \; a \leq b$

$$\Rightarrow a = \inf\{a, b\}$$

$$\Rightarrow a = a \wedge b \quad \{\because \inf\{a, b\} = a \wedge b$$

Proved

* Another definition of lattice →

A non empty set L is said to be lattice under binary operation ∨ and ∧ If it satisfing the following Property.

1- Commutative Property →

let $a, b \in L$ Then

$$a \wedge b = b \wedge a \quad \text{and} \quad a \vee b = b \vee a$$

2- Associative Property →

let $a, b, c \in L$ Then

$$a \vee (b \vee c) = (a \vee b) \vee c \quad \text{and}$$

$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

3→ Absorption Property →

Let $a, b \in L$ Then

$$a \vee (a \wedge b) = a \quad \text{and} \quad a \wedge (a \vee b) = a$$

* Sub lattices → 2001

P-128

A non empty subset S of lattice L is said to be sub-lattice of L If S is closed under operation ∨ and ∧. i.e

$$a \in S, b \in S \Rightarrow a \wedge b \in S \text{ and } a \vee b \in S$$

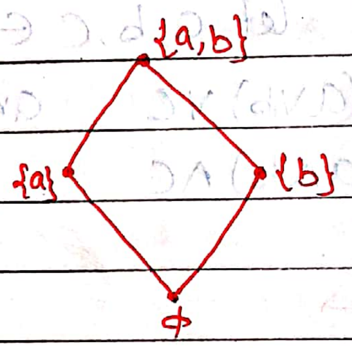
OR

A non empty subset S of lattices L is said to be sublattice of L If operation in both is same and for this operation S itself form the lattices.

Exp-3.15 Let $S = \{a, b, c\}$ and $P(S)$ be the Power set of S Then show $R = \{\phi, \{a\}, \{b\}, \{a,b\}\}$ is sublattice and $U = \{\phi, \{a\}, \{b\}\}$ is not a sublattice.

Proof

formed Hasse diagram for R $\longrightarrow$



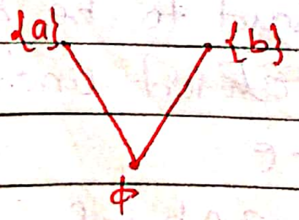
$\{a\} \in R$, $\{b\} \in R$

$\Rightarrow a \wedge b \in R \Rightarrow \phi \in R$

and $a \vee b = \{a,b\} \in R$

= Hence each element in R has joint and meet. So It satisfy the closure Property Then R is sublattice.

formed Hasse diagram for U $\longrightarrow$



As $\{a\} \in U$ $\{b\} \in U$

$a \wedge b = \phi \in U$ But

$a \vee b = \{a,b\} \notin U$ does not exist.

So U is not sub lattices.

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§3.6 Isomorphic lattices →

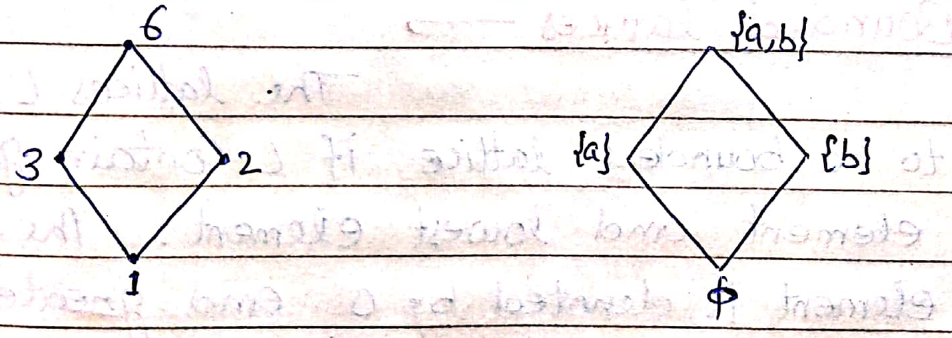
Let L_1, L_2 be two lattice then L_1 is said to be isomorphic to L_2 if we defind a mapping $f: L_1 \rightarrow L_2$ s.t

1. f is one-one
2. f is onto
3. mapping is Preserved $f(a \wedge b) = f(a) \wedge f(b)$
   $f(a \vee b) = f(a) \vee f(b)$

Exp- 3.22 Consider the lattice $L = \{1, 2, 3, 6\}$ under divisibility relation and the lattices $(P(S), \subseteq)$ where $S = \{a, b\}$. Then the lattices L and $P(S)$ are isomorphic.

Sol^n

formed Hasse diagram of L and $P(S)$ →



We defind the mapping $f: L \rightarrow P(S)$

s.t $\quad x \in L \Rightarrow f(x) \in P(S)$

i.e

$1 \in L \Rightarrow f(1) = \phi \in P(S)$

$2 \in L \Rightarrow f(2) = \{b\} \in P(S)$

$3 \in L \Rightarrow f(3) = \{a\} \in P(S)$

$6 \in L \Rightarrow f(6) = \{a, b\} \in P(S)$

Then it is clearly one-one and onto and mapping is Preserved

$$f(a \wedge b) = f(a) \wedge f(b)$$
$$f(a \vee b) = f(a) \vee f(b) \quad \forall a, b \in L$$

Let ~~aaaa~~ $a = 2, \; b = 3$

L.H.S $\quad f(2 \wedge 3) = f(1) = \phi$

R.H.S $\quad f(2) \wedge f(3) = \{b\} \wedge \{a\} = \phi$

Hence L.H.S = R.H.S

∴ Hence mapping is isomorphic.

* Bounded lattices →

The lattices L is said to be bounded lattice if L Contain greatest element and lowest element. The lowest element is denoted by 0 and greatest element is denoted by 1.