2001

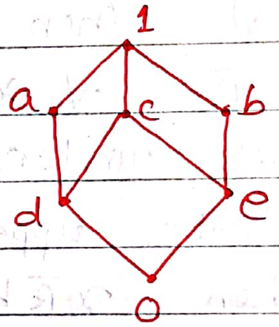
§3.8 Complements, Complemented lattices →

Let L be the bounded lattices with upper bound 1 and lower bound 0 and $a \in L$ also b is element in L. Then b is said to be Complement of a and a is said to be Complement of b if $a \vee b = 1$

$a \wedge b = 0$

A bounded lattice L is said to be Complemented lattice if every element in L has a Complement.

Exp-3.28
P-142



be the lattice Then find Complement of b and c.

Soln

If a is Complement of b Then $a \vee b = 1$ and $a \wedge b = 0$

i.e $a \vee b = 1$ and $a \wedge b = 0$

Hence a is Complement of b.

Also

$e \wedge a = 0$, $e \vee a = 1 \Rightarrow e$ is the complement of a.

C is not Complement of b as

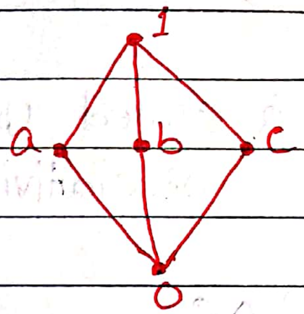
$b \vee c = 1$ But $b \wedge c \neq 0 = e$

29, 2021

§3.10 **Modular lattices** $\rightarrow$ A lattice L is said to be modular if for all $a, b, c \in L$ such that

$$a \leq c \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$$

Exp-3.36 Show that lattice



is modular lattices.

Soln If It is modular then

$$a \leq c \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$$

L.H.S $\quad a \vee (b \wedge c)$

$$= a \vee 0 = 1$$

R.H.S $\quad (a \vee b) \wedge c$

$$= 1 \wedge c = 0$$

Hence L.H.S $\neq$ R.H.S

So It is not modular lattices.

Theorem-1 A sublattice of a modular lattice is modular.

P-152

Proof

Let S is sublattice of Modular lattice L. Since L is modular and $a,b,c \in L$ Then

$$a \vee (b \wedge c) = (a \vee b) \wedge c \quad \text{—— (1)}$$

Since $S \subset L$ and S is closed under $\vee$ and $\wedge$. Then It follow associativity

i.e $\quad a \vee (b \wedge c) = (a \vee b) \wedge c$

$\Rightarrow S$ is also modular.

Theorem-3 Dual of modular lattice is modular.

Proof

Let $(L, \leq)$ be the modular lattice Then show It dual $(L, \geq)$ is also a modular lattice.

Let $a, b, c \in L$ and let $a \geq c$ Then show

$$a \wedge (b \vee c) = (a \wedge b) \vee c \quad \text{—— (1)}$$

Since $(L, \leq)$ is modular and $c \leq a$ Then

$$c \vee (b \wedge a) = (c \vee b) \wedge a$$

$$\Rightarrow c \vee (a \wedge b) = (b \vee c) \wedge a$$

$$\Rightarrow (a \wedge b) \vee c = a \wedge (b \vee c)$$

Hence $(L, \geq)$ is modular.

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* Distributive lattices →

A lattices L is said to be distributive lattice if $a, b, c \in L$ and

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \quad \text{—— (1)}$$

OR $$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \quad \text{—— (2)}$$

Exp- 4.43
P-160

Every chain is distributive lattices.

Sol^n

Let L is the chain of lattices and $a, b, c \in L$.

Since L is chain Then any two element of a, b and c must be Comparable. ~~there~~ There are two Possiblity.

(i) $a \leq b$ or $a \leq c$

(ii) $a \geq b$ or $a \geq c$

If $a \leq b$ OR $a \leq c$ Then show

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

L.H.S $\quad a \wedge (b \vee c) = a$

R.H.S $\quad (a \wedge b) \vee (a \wedge c) = a$

$\Rightarrow$ L.H.S = R.H.S

Hence it is distributive lattices.