Date 08/04/20

Lecture notes – 15

BCA I year (II Sem)

Subject - Mathematics - II (UNIT- 5)

Topic - Equation of plane

Exp-13 Find the equation to the plane through the points (0,0,2) (1,2,1) and (3,1,0).

Solution We know that the equation of plane passing through the points (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) is given by

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

So the equation of plane passing through the points (0,0,2), (1,2,1) and (3,1,0) is

$$\begin{vmatrix} x & y & z & 1 \\ 0 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{vmatrix} = 0$$

Or

$$\begin{vmatrix} x & y & z & 1 \\ -x & -y & 2-z & 0 \\ 1-x & 2-y & 1-z & 0 \\ 3-x & 1-y & -z & 0 \end{vmatrix} = 0$$

Scanned with CamScanner

Expand about Column 4th, then

$$\begin{vmatrix} -1 & -x & -y & 2-z \\ & 1-x & 2-y & 1-z \\ & 3-x & 1-y & -z \end{vmatrix} = 0$$

Now expand through Row 1st, then

$$\Rightarrow -\left[(-x)\{(2-y)(-z)-(1-y)(1-z)\}-(-y)\{(1-x)(-z)-(3-x)(1-z)\}\right.$$
$$\left.+(2-z)\{(1-x)(1-y)-(3-x)(2-y)\}\right]$$

$$\Rightarrow -\left[(-x)\{-2z+yz-1+z+y-yz\}+y\{-z+xz-3+3z+x-xz\}\right.$$
$$\left.+(2-z)\{-y+1-x+xy-6+3y+2x-xy\}\right]$$

$$\Rightarrow -\left[(-x)\{-2z-1+z+y\}+y\{+2z-3+x\}+\right.$$
$$\left.(2-z)\{2y-5+x+xy\}\right]$$

NOT

$$\Rightarrow -\left[2xz+x-xz-xy+2yz-3y+xy+\right.$$
$$\left.+4y-10+2x+xy-2yz+5z-xz-xy\right]$$

$$\Rightarrow -[3x+y+5z-10]=0$$

$$\Rightarrow 3x+y+5z+10=0$$

The required equation of the plane.

Scanned with CamScanner

Exp-15 Show that the points $(0,0,2)$, $(1,2,1)$, $(3,1,0)$ and $(-2,1,-2)$ are Coplanar.

From Exp-13.

The equation of plane passing through the points $(0,0,2)$, $(1,2,1)$ and $(3,1,0)$ is

$$3x+y+5z-10=0$$

$\because$ point $(-2,1,-2)$ does not satisfy the Equation of plane.

So these four points are not Coplanar.

NOTE If points are $(2,-1,1)$.

Then point $(2,-1,1)$ satisfies the Equation of plane.

Ans. $3(2)+(-1)+5(2)-10$

$\Rightarrow 6-1+5-10 \Rightarrow 5+5-10=0$

Hence points $(0,0,2)$, $(1,2,1)$, $(3,1,0)$ and $(2,-1,1)$ are Coplanar.

Scanned with CamScanner

II Method

Exp-16 Show that the four points $(0,-1,-1)$ $(4,5,1)$ $(3,9,4)$ and $(-4,4,4)$ are Coplanar.

Solⁿ **II method**

The equation of plane passing through three points is

$$Ax + By + Cz + D = 0 \quad \text{---(i)}$$

If Equation (i) passes through the point $(4,5,1)$.

$$\Rightarrow 4A + 5B + C + D = 0 \quad \text{---(ii)}$$

Similarly, if Equation (i) passes through the points $(3,9,4)$ and $(-4,4,4)$

$$\Rightarrow 3A + 9B + 4C + D = 0 \quad \text{---(iii)}$$

and $$-4A + 4B + 4C + D = 0 \quad \text{---(iv)}$$

Solve Equation (ii), (iii) and (iv) Simultaneously.

From Eqⁿ (iii) & (iv)

$$\begin{array}{r} 3A + 9B + 4C + D = 0 \\ -4A + 4B + 4C + D = 0 \\ \hline 7A + 5B = 0 \end{array}$$

Or $$\boxed{A = -\frac{5}{7}B} \Rightarrow B = -\frac{7A}{5}$$

Scanned with CamScanner

Similarly, $C = -\frac{11}{7}B$

and $D = -\frac{4}{7}B$.

Putting these values in Eqn (i)

$\Rightarrow -\frac{5}{7}Bx + By - \frac{11}{7}Bz - \frac{4}{7}B = 0$

$\Rightarrow 5x - 7y + 11z + 4 = 0$ ——— (V)

and the fourth point $(0, -1, -1)$ passes through Equation (V).

Thus the given four points are Coplanar.

NOTE We Can use any one of these two methods for finding the equation of plane.

Exp-17 Find the direction ratios and direction Cosines of the plane passing through the points $(-1, 4, 3)$, $(3, 2, -5)$, $(-3, 8, -5)$ and $(-3, 2, 1)$.

Do itself

Scanned with CamScanner