Date
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Lecture notes - 16
BCA I year (II Sem)
Sub - mathematics - II
Topic - Equation of plane

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UNIT-5

Solution Equation of plane passing throug the points $(-1,4,-3)$ $(3,2,-5)$ $(-3,8,-5)$ is

$$\begin{vmatrix} x & y & z & 1 \\ -1 & 4 & -3 & 1 \\ 3 & 2 & -5 & 1 \\ -3 & 8 & -5 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y & z & 1 \\ -1-x & 4-y & -3-z & 0 \\ 3-x & 2-y & -5-z & 0 \\ -3-x & 8-y & -5-z & 0 \end{vmatrix} = 0$$

Expand through Column 4th

$$\Rightarrow -1 \begin{vmatrix} -1-x & 4-y & -3-z \\ 3-x & 2-y & -5-z \\ -3-x & 8-y & -5-z \end{vmatrix} = 0$$

$R_3 \to R_3 - R_2$

$$\Rightarrow -1 \begin{vmatrix} -1-x & 4-y & -3-z \\ +3-x & 2-y & -5-z \\ -6 & 6 & 0 \end{vmatrix} = 0$$

Now Expand through Row 3rd.

$$\Rightarrow -1\left[-6\{(4-y)(-5-z)-(2-y)(-3-z)\}-6\{(-1-x)(-5-z)-(3-x)(-3-z)\}+0\right]=0$$

$$\Rightarrow 1\left[-6\{-20-4z+5y+yz+6+2z-3y-yz\}\right.$$
$$\left.-6\{5+z+5x+xz+9+3z-3x-xz\}\right]=0$$

$$\Rightarrow -1\left[-6(-14-2z+2y)-6(14+4z+2x)\right]=0$$

$$\Rightarrow -1\left[84+12z-12y-84-24z-12x\right]=0$$

$$\Rightarrow -1\left[-12x-12y-12z\right]=0$$

$$\Rightarrow 12x+12y+12z=0$$

Or $x+y+z=0$

This is the equation of required plane

∵ The point $(-3,2,1)$ satisfy the given Equation

So $x+y+z=0$ passes through $(-3,2,1)$

Hence given four points are Coplanar.

The direction ratios of plane are the Coefficients of x, y, z then d.r's are $(1,1,1)$.

and the required direction Cosines are

$$\frac{1}{\sqrt{(1)^2+(1)^2+(1)^2}}, \frac{1}{\sqrt{1+1+1}}, \frac{1}{\sqrt{1+1+1}}$$

$$\Rightarrow \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \quad \text{Ans}$$

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Ques-1 Find the equation of plane passing through the points (0,0,-1) (0,1,0) and (2,3,0)

Ques-2 Prove that the points (0,-1,-1) (4,5,1) (3,9,4) and (-4,4,4) are Coplanar.

Solve itself

* Angles between two planes →

If $A_1x + B_1y + C_1z + D_1 = 0$ and

$A_2x + B_2y + C_2z + D_2 = 0$ are two planes then angle between two planes is

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\ \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

NOTE If $A_1A_2 + B_1B_2 + C_1C_2 = 0$ Then two Plane are perpendicular.

NOTE If $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ Then two Planes are parallel.

Exp-18 Find the angle between the plane $x+y+z+1=0$ and $2x-y-z+5=0$.

Solution

Given planes are

$$x+y+z+1=0$$

$$2x-y-z+5=0$$

Then direction ratios of the planes are $(1,1,1)$ and $(2,-1,-1)$

Then

$$\cos\theta = \frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\ \sqrt{A_2^2+B_2^2+C_2^2}}$$

$$\Rightarrow \cos\theta = \frac{(1)(2)+(1)(-1)+(1)(-1)}{\sqrt{(1)^2+(1)^2+(1)^2}\ \sqrt{(2)^2+(1)^2+(-1)^2}}$$

$$\Rightarrow \cos\theta = \frac{2-1-1}{\sqrt{1+1+1}\ \sqrt{4+1+1}}$$

$$\Rightarrow \cos\theta = \frac{0}{\sqrt{3}\sqrt{6}} = 0$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^0$$

Hence the planes are perpendicular to each other.