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Lecture Notes- 19

BCA I Year (II Sem)

Subject - Mathematics - II

Topic - Equation of plane Unit-5

Exp-26 Show that the equation
$2x^2-6y^2-12z^2+18yz+2zx+xy=0$
represents a pair of planes and find the angle between them.

Solution We have $2x^2-6y^2-12z^2+18yz+2zx+xy=0$

Compairing this equation

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$$

We get $a=2,\ b=-6,\ c=-12,\ f=9,\ g=1,\ h=\frac{1}{2}$

$\therefore\ abc+2fgh-af^2-bg^2-ch^2$

$= 2(-6)+(-12)+2(9)(1)(\frac{1}{2})-2(81)+6(1)+12(\frac{1}{4})$

$= 144+9-162+6+3=0$

Hence the given equation represents a Pair of plane

If θ is the angle between the planes then

$$\tan\theta=\frac{2\sqrt{f^2+g^2+h^2-ab-bc-ca}}{a+b+c}$$

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$$\Rightarrow \tan\theta = \frac{2\sqrt{81+1+\frac{1}{4}+12-72+24}}{2-6-12}$$

$$\Rightarrow \tan\theta = \frac{-\sqrt{185}}{16}$$

$$\because \sec^2\theta = 1+\tan^2\theta = 1+\frac{185}{16} = \frac{441}{256}$$

$$\Rightarrow \sec\theta = \frac{21}{16} \Rightarrow$$

$$\Rightarrow \cos\theta = \frac{16}{21} \Rightarrow \boxed{\theta = \cos^{-1}\frac{16}{21}}$$

Ans

Imp

Exp 27 Find the area of the triangle whose vertices are $A(1,2,3)$, $B(2,-1,1)$, $C(1,2,-4)$

Solution Let $\Delta_x, \Delta_y, \Delta_z$ are the area of the projection of the area of ΔABC on xy, yz, zx plane, then

$$\Delta_x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 2 & -4 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta_x = \frac{1}{2}\left[2(5)-3(-3)+1(2)\right]$$

$$\Rightarrow \Delta_x = \frac{1}{2}\left[10+9+2\right] = \frac{21}{2}$$

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$$\Delta_y = \frac{1}{2}\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & -4 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta_y = \frac{1}{2}\left[1(5) - 3(1) + 1(-9)\right]$$

$$\Rightarrow \Delta_y = \frac{1}{2}\left[5 - 3 - 9\right] = -\frac{7}{2}$$

$$\Delta_z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Delta_z = \frac{1}{2}\left[1(-3) - 2(1) + 1(5)\right]$$

$$\Delta_z = \frac{1}{2}\left[-3 - 2 + 5\right] = 0$$

Hence the Area of ΔABC is

$$\Delta = \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$$

$$\Rightarrow \Delta = \sqrt{\left(\frac{21}{2}\right)^2 + \left(-\frac{7}{2}\right)^2 + (0)^2}$$

$$\Delta = \sqrt{\frac{441}{4} + \frac{49}{4} + 0}$$

$$\Rightarrow \Delta = \frac{1}{2}\sqrt{490} \quad \text{Ans}$$