

Date
16-04-20

Lecture notes - 21

BCA I Year (II Sem)

PAGE _____

DATE ____/____/____

Mathematics - II (Unit - 5)

Topic - Straight line

* The equation of straight line in general form is $ax + by + cz + d = 0$

* The equation of straight line in symmetrical form is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

Case-I The equation of straight line passing through (α, β, γ) and having direction cosines proportional to a, b, c is

$$\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c}$$

* Equation of straight line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Exp-1 ^{Imp} find where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the plane $2x+4y-z+1=0$

Solution Given $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} = r$ (Let) ①

and given plane $2x+4y-z+1=0$ ②

Then any point P on the line is

$$(2r+1, -3r+2, 4r-3)$$

If line intersect the given plane at point P then

$$2(2r+1) + 4(-3r+2) - (4r-3) + 1 = 0$$

$$\Rightarrow 4r+2 - 12r+8 - 4r+3 + 1 = 0$$

$$\Rightarrow 12r+14 = 0 \Rightarrow r = \frac{14}{12} = \frac{7}{6}$$

$$\text{Hence } \left[2\left(\frac{7}{6}\right)+1, -3\left(\frac{7}{6}\right)+2, 4\left(\frac{7}{6}-3\right) \right]$$

$$\Rightarrow \left[\frac{10}{3}, -\frac{3}{2}, \frac{5}{3} \right] \text{ Ans}$$

Exp-2 ^{Imp} Find the Coordinates of the point where the line joining the points $(2, -3, 1)$ and $(3, -4, -5)$ meets the plane $2x+y+z=7$

Solution The equation of straight line passing through two points is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = \lambda$$

∴ points are $(2, -3, 1)$ and $(3, -4, 5)$

$$\Rightarrow \frac{x-2}{3-2} = \frac{y+3}{-4+3} = \frac{z-1}{-5-1} = \lambda$$

$$\Rightarrow \frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \lambda$$

$$\Rightarrow x = \lambda + 2, \quad y = -\lambda - 3, \quad z = -6\lambda + 1$$

$$\text{then } P(x, y, z) = (\lambda + 2, -\lambda - 3, -6\lambda + 1)$$

If the point lies on the given plane

$$\Rightarrow 2(\lambda + 2) + (-\lambda - 3) + (-6\lambda + 1) = 7$$

$$\Rightarrow 2\lambda + 4 - \lambda - 3 - 6\lambda + 1 = 7$$

$$\Rightarrow -5\lambda + 2 = 7$$

$$\Rightarrow -5\lambda = 5 \Rightarrow \boxed{\lambda = -1}$$

Then required points are

$$x = -1 + 2 = 1, \quad y = +1 - 3 = -2, \quad z = -6(-1) + 1 = 7$$

$$P(x, y, z) = (1, -2, 7) \quad \text{Ans}$$