Date 21-04-20

Lecture Notes - 24

BCA 1 Year (II Sem)

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Subject - Mathematics - II (Unit-5)

Topic - Straight line

Exp-25 Find the length of the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Also show that it equation are given by

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Solution the equation of given line are

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = r_1 \quad \text{---- (i)}$$

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = r_2 \quad \text{---- (ii)}$$

Then Any point P on line (i) is

$$(r_1+3, -2r_1+5, r_1+7) \quad \text{---- (3)}$$

and any point on line (ii) is

$$(7r_2-1, -6r_2-1, r_2-1) \quad \text{---- (4)}$$

The direction ~~Cosines~~ ratios of line PQ are

$$(7r_2-1)-(r_1+3), (-6r_2-1)-(-2r_1+5), (r_2-1)-(r_1+7)$$

$$\Rightarrow 7r_2-r_1-4, -6r_2+2r_1-6, r_2-r_1-8$$

---- (5)

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Let the line PQ be the line of S.D. So that PQ is perpendicular to both given line ① and ② then

$1(7r_2 - r_1 - 4) - 2(-6r_2 + 2r_1 - 6) + 1(r_2 - r_1 - 8) = 0$

and $7(7r_2 - r_1 - 4) - 6(-6r_2 + 2r_1 - 6) + 1(r_2 - r_1 - 8) = 0$

Or $20r_2 - 6r_1 = 0$

and $86r_2 - 20r_1 = 0$

by Solving these equations we get

$r_1 = r_2 = 0$

Put the value of r_1 and r_2 in equation ③, ④ and ⑤ Then we have

$P(3, 5, 7)$ and $Q(-1, -1, -1)$

and the direction ratios of PQ are $(-4, -6, -8)$ or $(2, 3, 4)$

Hence the length of PQ

$$= \sqrt{(-1-3)^2 + (-1-5)^2 + (-1-7)^2}$$

$$= \sqrt{(4)^2 + (-6)^2 + (-8)^2}$$

$$= \sqrt{16 + 36 + 64} = \sqrt{116}$$ Ans

Ans