Date
23-04-20

Lecture notes - 26

BCA I Year (II Sem)

Subject -: Mathematics - II (Unit-5)

Topic - Sphere

Exp-10 Find the equation of the Sphere with Centre $(2,3,-4)$ and touching the plane $2x+6y-3z+15=0$

Solution Given Centre of the Sphere is $(2,3,-4)$

$\because$ Sphere touching the plane

$\therefore$ Radius of the Sphere = lenght of the perpendicular from the Centre $(2,3,-4)$ on the plane $2x+6y-3z+15=0$

$$\text{Radius} = \frac{2(2)+6(3)-3(-4)+15}{\sqrt{(2)^2+(6)^2+(-3)^2}} = \frac{49}{7} = 7$$

So the required equation of Sphere

$$\Rightarrow (x-2)^2+(y-3)^2+(z+4)^2=(7)^2$$

$$\Rightarrow x^2+y^2+z^2-4x-6y+8z-20=0$$

Ans

* **Intersection of two Spheres →**

* **The System of Spheres through a given Circle →**

Exp-12 Find the equation to the Sphere through the Circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point $(1,2,3)$

Solution Given $S = x^2+y^2+z^2-9=0$ is Sphere

and $P = 2x+3y+4z-5=0$ is plane

Then any Sphere passing through given Circle

$$S+\lambda P=0$$

$$\Rightarrow (x^2+y^2+z^2-9)+\lambda(2x+3y+4z-5)=0$$

It passes through $(1,2,3)$

$$\Rightarrow \left[(1)^2+(2)^2+(3)^2-9\right]+\lambda\left[2(1)+3(2)+4(3)-5\right]=0$$

$$\Rightarrow (1+4+9-9)+\lambda(2+6+12-5)=0$$

$$\Rightarrow 5+15\lambda=0 \Rightarrow \lambda=\frac{-1}{3}$$

Then required equation of Plane is

$$(x^2+y^2+z^2-9)+\frac{1}{3}(2x+3y+4z-5)=0$$

$$\Rightarrow 3x^2+3y^2+3z^2-27+2x+3y+4z-5=0$$

$$\Rightarrow 3x^2+3y^2+3z^2+2x+3y+4z-32=0$$

Ans

Exp-13 Find the equation of Sphere which passes through the point (α, β, γ) and the Circle $x^2+y^2=a^2$, $z=0$

Solution Let $S = x^2+y^2-a^2=0$ is given Sphere and $P = z=0$ is plane.

Then any Sphere passing through given Circle is

$$S+\lambda P = 0$$

$$\Rightarrow (x^2+y^2-a^2) + \lambda(z) = 0$$

$\because$ The Sphere passes through the point (α, β, γ)

$$\Rightarrow (\alpha^2+\beta^2+\cancel{a^2}) + \lambda(\gamma) = 0$$

$$\Rightarrow \lambda = -\frac{(\alpha^2+\beta^2\cancel{+a^2})}{\gamma}$$

Then the required equation of Sphere is

$$\Rightarrow (x^2+y^2-a^2) - \frac{(\alpha^2+\beta^2-a^2)}{\cancel{\gamma}}\cancel{(\gamma)} = 0$$

$$\Rightarrow x^2+y^2 \cancel{-a^2} + \alpha^2+\beta^2 \cancel{+a^2} = 0$$

$$\Rightarrow x^2+y^2+\alpha^2+\beta^2 = 0$$

Ans