$>$ The implementation of half adder using exclusive-OR and an AND gates is used to show that two half adders can be used to construct a full adder.
$>$ The inputs to the $\mathbf{X O R}$ gate are also the inputs to the AND gate.

## 2. Full Adder

Full Adder is a combinational circuit that performs the addition of three bits (two significant bits and previous carry).
$>$ It consists of three inputs and two outputs, two inputs are the bits to be added, the third input represents the carry form the previous position.
$\rightarrow$ The full adder is usually a component in a cascade of adders, which add 8,16 , etc, binary numbers.

| Inputs |  |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $X$ | $Y$ | $C_{\text {in }}$ | $S$ | $C_{\text {out }}$ |  |
| 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 1 | 1 | 0 |  |
| 0 | 1 | 0 | 1 | 0 |  |
| 0 | 1 | 1 | 0 | 1 |  |
| 1 | 0 | 0 | 1 | 0 |  |
| 1 | 0 | 1 | 0 | 1 |  |
| 1 | 1 | 0 | 0 | 1 |  |
| 1 | 1 | 1 | 1 | 1 |  |
| Truth table for the full adder |  |  |  |  |  |

> The $S$ output is equal to 1 when only one input is equal to 1 or when all three inputs are equal to 1 .
$>$ The $C_{\text {out }}$ output has a carry 1 if two or three inputs are equal to 1 .
$>$ The Karnaugh maps and the simplified expression are shown in the following figures:


$$
\left\{\begin{array}{ll}
\mathbf{S}=\overline{\mathbf{X}} \overline{\mathbf{Y}} \mathbf{C}_{\mathbf{i n}}+\overline{\mathbf{X}} \mathbf{Y} \overline{\mathbf{C}_{\mathbf{i n}}}+\mathbf{X} \overline{\mathbf{Y}} \overline{\mathbf{C}_{\mathbf{i n}}}+\mathbf{X Y \mathbf { C } _ { \mathbf { i n } }} & \\
\mathbf{C}_{\mathbf{o u t}} \mathbf{X Y}+\mathbf{X C} \mathbf{C}_{\mathbf{i n}}+\mathbf{Y} \mathbf{C}_{\mathbf{i n}} & \mathbf{1}
\end{array}\right\} \text { (Sum of products) }
$$

