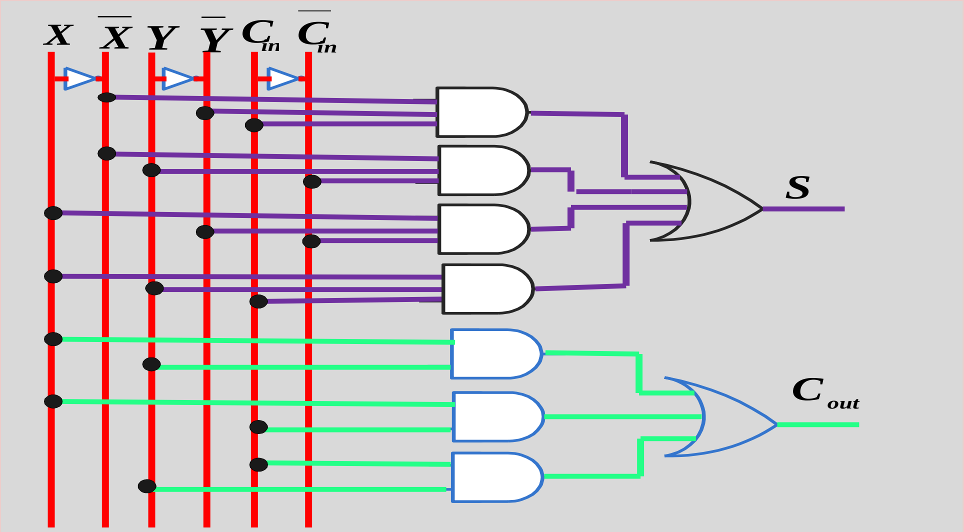
- The ***logic diagrams*** for the full adder implemented in ***sum-of-products*** form are the following:



logic diagram for the full adder

- It can also be implemented using ***two half adders*** and ***one OR gate*** (using **XOR** gates).

$$\begin{cases} S = C_{in} \oplus (X \oplus Y) \\ C_{out} = C_{in} \cdot (X \oplus Y) + XY \end{cases}$$

Proof:

The sum:

$$\mathbf{S = \overline{X}\,\overline{Y}C_{in} + \overline{X}Y\overline{C_{in}} + X\overline{Y}\,\overline{C_{in}} + XYC_{in}}$$

$$\mathbf{= \overline{C_{in}}(\overline{X}Y + X\overline{Y}) + C_{in}(\overline{X}\,\overline{Y} + XY)}$$

$$\mathbf{= \overline{C_{in}}(\overline{X}Y + X\overline{Y}) + C_{in}\overline{(\overline{X}Y + X\overline{Y})}}$$

$$S = C_{in} \oplus (X \oplus Y)$$

The carry output:

$$C_{out} = \mathbf{\overline{X}YC_{in} + X\overline{Y}C_{in} + XYC_{in} + XY\,\overline{C_{in}}}$$

$$\mathbf{= C_{in}(\overline{X}Y + X\overline{Y}) + XY(C_{in} + \overline{C_{in}})}$$

$$C_{out} = C_{in} \cdot (X \oplus Y) + XY$$