LATTICES

CHAPTER-3

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98, 99, 2002

Lattices ⟶ The lattices $(L, \leq)$ is a Poset if for every $a, b \in L$ Then $Sup.\{a,b\}$ and $\inf\{a,b\}$ must be exist in L and we defind suprimum and infimum of $\{a,b\}$ as

$$Sup\{a,b\} = a \vee b \quad \text{i.e } a \text{ joint } b$$

$$\inf\{a,b\} = a \wedge b \quad \text{i.e } a \text{ meets } b$$

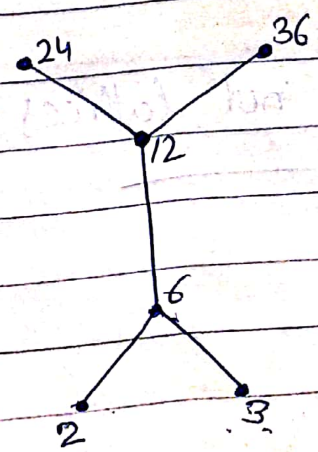
If A, B be two sets in L Then

$$Sup\{A,B\} = A \vee B \quad OR \quad A \cup B \quad OR \quad A+B$$

$$\inf\{A,B\} = A \wedge B \quad OR \quad A \cap B \quad OR \quad AB$$

Ques Let $X = \{2, 3, 6, 12, 24, 36\}$ is Poset Then show It is not lattices and Relation is divisiblity.

Soln first find the Hasse diagram



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X will be Poset if

1- Reflexive $\longrightarrow$ we know each element is divisible by itself.

i.e $2|2$, $3|3$

OR aRa $\forall a \in X$

2- Anti-Symmetric $\longrightarrow$ If $a|b$ and $b|a$ Then $a=b$

Hence $2|6$ Then 6 Can be divides 2 iff $2=6$ This is anti-Symmetric.

3- Transitive $\longrightarrow$

$$a|b,\ b|c \Rightarrow a|c$$

as

$$2|6,\ 6|12 \Rightarrow 2|12$$

Hence X is Poset.

Now to show X is not lattices.

Let $a, b \in X$ OR $(2,3) \in X$

$$Sup\{2,3\} = 2 \vee 3 = 6 \in X.$$

$$\inf\{2,3\} = 2 \wedge 3$$ does not exist.

Then X is not lattices

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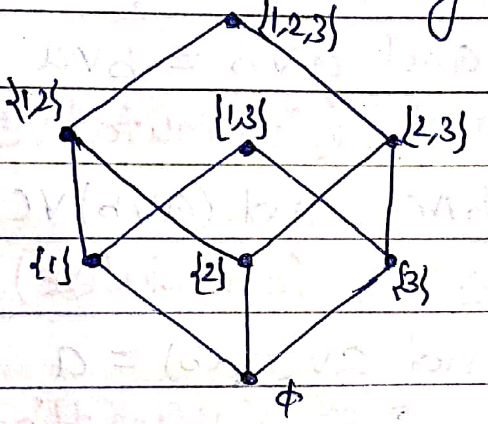
Ques $S = \{1, 2, 3\}$

$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

be subset of S. Then show
P(S) is lattices.

Sol^n

formed Hasse digram



This Poset is lattices as each element of S has sup. and inf.

$\sup\{\{1\}, \{2\}\} = \{1\} \vee \{2\} = \{1,2\} = \{1\} \cup \{2\}$

$\inf\{\{1\}, \{2\}\} = \{1\} \wedge \{2\} = \phi = \{1\} \cap \{2\}$

Hence P(S) is lattices.

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Some Properties of lattices

2004

Theorem-1 P-116 If $(L, \leq)$ is a lattices Then from any $a, b, c \in L$ Then

1- $a \wedge a = a$ and $a \vee a = a$ (Idempotent Law)

2- $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$ (Commutative)

3- $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $(a \vee b) \vee c = a \vee (b \vee c)$ (Associative)

4- $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$ (Absorption)

Proof

1→ $a \wedge a = \inf\{a, a\} = a$

$\Rightarrow a \wedge a = a$

$a \vee b = \sup\{a, a\} = a$

$\Rightarrow a \vee a = a$

2- $a \wedge b = b \wedge a$

L.H.S

$a \wedge b = \inf\{a, b\} = \inf\{b, a\}$

$= b \wedge a$

$\Rightarrow a \wedge b = b \wedge a$

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$a \vee b = b \vee a$

LHS $\quad a \vee b = \sup\{a,b\} = \sup\{b,a\}$
$= b \vee a$

$\Rightarrow a \vee b = b \vee a$

3→ $(a \wedge b) \wedge c = a \wedge (b \wedge c)$

let $b \wedge c = d$ Then
$d = \inf\{b,c\}$
$\Rightarrow d \leq b, \; d \leq c$

Now let $a \wedge d = e$ Then
$e = \inf\{a,d\}$
$\Rightarrow e \leq a, \; e \leq d$

as

$e \leq d$ and $d \leq b$
$\Rightarrow e \leq b$

$\therefore e \leq a, \; e \leq b$ and $d \leq c, d \leq b$
$\Rightarrow e$ is the lower bound of $\{a,b,c\}$

If f is any lower bound of $\{a,b,c\}$

$f \leq a, \; f \leq b, \; f \leq c$

as $f \leq b$, $b \leq c$ and $d = \inf\{b, c\}$

$\Rightarrow f \leq d$

and $f \leq a$ and $e = \inf\{a, d\}$

$\Rightarrow f \leq e$

Hence $e = \inf\{a, b, c\} = a \wedge (b \wedge c)$

Similarly $e = \inf\{a, b, c\} = (a \wedge b) \wedge c$

Thus $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ Proved

4→ $a \wedge (a \vee b) = a$

L.H.S we know $a \leq a$ and $a \leq a \vee b$

$\Rightarrow a \leq a \wedge (a \vee b)$ —①

Again $a \wedge (a \vee b) = \inf\{a, a \vee b\} \leq a$ —②

from Eqn ② & ①

$a \wedge (a \vee b) = a$

Proved