

UNIT-IV TOPIC-LATTICES* Conditional statement OR If Part ORNecessary Condition OR Then \rightarrow

Let P and q be two statements. Then P Conditional q is denoted by $P \Rightarrow q$ OR $P \rightarrow q$ OR P then q and defined as

If P is true and q is false Then $P \rightarrow q$ is false otherwise true

Truth table for $P \rightarrow q \Leftrightarrow$

P	q	$P \rightarrow q$
T	F	F
T	T	T
F	T	T
F	F	T

* Bi Conditional Statement \rightarrow

Let P and q be two statements. Then P biConditional q is denoted by $P \Leftrightarrow q$ OR $P \leftrightarrow q$ and defined as

If P and q both have same value Then $P \leftrightarrow q$ is true otherwise false.

Truth table for $P \leftrightarrow q \rightarrow$

P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

v. Imp 2004

* Tautology and Contradiction \rightarrow

A statement which is true for all possible values then it is said to be tautology.

A statement which is always false for all possible values then it is called contradiction.

with example.

Example Show that $[P \wedge (P \rightarrow q)] \rightarrow q$ is Tautology.

Sol ⁿ	P	q	$P \rightarrow q$	$P \wedge (P \rightarrow q)$	$[P \wedge (P \rightarrow q)] \rightarrow q$
	T	T	T	T	T
	T	F	F	F	T
	F	T	T	F	T
	F	F	T	F	T

Since $[P \wedge (P \rightarrow q)] \rightarrow q$ contains only T then it is tautology.

* Equivalent statements OR functions →

Two statements P and Q are said to be logically equivalent if they have same or identical truth value. It is denoted by '='.

Example Prove that $P \rightarrow Q = \sim P \vee Q$

<u>Solⁿ</u>	P	Q	$P \rightarrow Q$	$\sim P$	$\sim P \vee Q$
	T	T	T	F	T
	T	F	F	F	F
	F	T	T	T	T
	F	F	T	T	T

we observed that the truth values of $P \rightarrow Q$ and $\sim P \vee Q$ are identical. Then

$P \rightarrow Q = \sim P \vee Q$ so these are equivalent statement.

* Law of Duality \rightarrow The duality is obtained by changing \vee and \wedge i.e

$(P \vee Q) \wedge R$ Then its dual is $(P \wedge Q) \vee R$

* Connectives \rightarrow (\downarrow and \uparrow) \rightarrow

There are two other connectives is called "NAND" and "NOR" Then NAND is denoted by \uparrow and NOR is denoted by \downarrow . and defined as

If P and Q are two statements
 Then $P \uparrow Q = \neg (P \wedge Q)$
 and $P \downarrow Q = \neg (P \vee Q)$

Truth table for 'NAND' and 'NOR' \rightarrow

P	Q	$P \wedge Q$	$P \vee Q$	$P \uparrow Q = \neg (P \wedge Q)$	$P \downarrow Q = \neg (P \vee Q)$
T	T	T	T	F	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

Arguments → An argument is a process by which a conclusion is formed from a given set of statements called Premises.

An argument is said to be valid argument iff the conjunction of the premises implies the conclusion. i.e $P_1, P_2, P_3 \dots P_n$ is valid iff the statement

$$(P_1 \wedge P_2 \wedge P_3 \dots \wedge P_n) \rightarrow R \text{ (truth)}$$

is tautology.

Example

Prove that the following argument is valid.

$$\left. \begin{array}{l} P \\ P \rightarrow q \end{array} \right\} \text{Premises}$$

$$q \rightarrow \text{Conclusion}$$

Solⁿ

The argument is valid iff

$$[P \wedge (P \rightarrow q)] \rightarrow q \text{ (true)}$$

OR tautology.

P	q	$P \rightarrow q$	$P \wedge (P \rightarrow q)$	$[P \wedge (P \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Then the statement is valid.