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BCA I year (II Sem)

Subject - Mathematics - II UNIT-6

Topic - Multiple Integral

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* **Double Integrals** →

$$\iint_A f(x,y)\,dA = \iint_A f(x)\cdot f(y)\,dx\cdot dy$$

It is called double integral of $f(x,y)$ over the region A.

* **properties of double integral** →

(i) If region A is divided into two parts A_1 and A_2 then

$$\iint_A f(x,y)\,dx\cdot dy = \iint_{A_1} f(x,y)\,dx\cdot dy + \iint_{A_2} f(x,y)\,dx\cdot dy$$

Similarly for sub-division of A into three and more parts.

(ii)
$$\iint_A m\cdot f(x,y)\,dx\cdot dy = m\iint_A f(x,y)\,dx\,dy$$

Where m is a Constant.

NOTE Let $\int_a^b \int_c^d f(x,y)\,dx\,dy$, where a, b, c, d are Constant.

If we integrate first integral ~~then~~ with respect to x then y taking as Constant When we integrate w.r. to y then x taking as Constant.

NOTE $\int_a^b \int_{f(x)}^{g(x)} f(x,y)\,dx\,dy$

If variable limit is in x then first integrate w.r. to y then w.r. to x.

NOTE $\int_a^b \int_{\phi(y)}^{g(y)} f(x,y)\,dx\,dy$

If variable limit is in y then first integrate w.r. to x and the w.r. to y.

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Exp-1 Evaluate $\int_1^2 \int_0^{3y} y \, dx \cdot dy$

Solution $\int_1^2 \int_0^{3y} y \, dx \, dy = \int_1^2 \left[\int_0^{3y} dx \right] y \, dy$

$\Rightarrow \int_1^2 [x]_0^{3y} \cdot y \, dy$

$\Rightarrow \int_1^2 (3y - 0) \cdot y \, dy$

$\Rightarrow \int_1^2 3y^2 dy = 3 \int_1^2 y^2 dy$

$\Rightarrow 3 \left[\frac{y^3}{3} \right]_1^2 = \cancel{3} \times \frac{1}{\cancel{3}} \left[y^3 \right]_1^2$

$\Rightarrow \left[(2)^3 - (1)^3 \right]$

$\Rightarrow \left[8 - 1 \right] = 7$ Ans

Exp-2 Evaluate $\int_0^1 \int_0^2 (x+y)\, dx\, dy$

Solution $\int_0^1 \int_0^2 (x+y)\, dx\, dy = \int_0^1 \left[\int_0^2 (x+y)\, dy\right] dx$

$$= \int_0^1 \left[x \cdot y + \frac{y^2}{2}\right]_0^2 dx$$

$$= \int_0^1 \left[2x + 2\right] dx$$

$$= \left[2\left(\frac{x^2}{2}\right) + 2x\right]_0^1$$

$$\Rightarrow \left[x^2 + 2x\right]_0^1$$

$$= \left[(1)^2 + 2(1) - (0) - (0)\right]$$

$$= 3 \quad \text{Ans}$$

Exp-3 $\int_2^3 \int_0^1 (x^2 + 3y^2)\, dx.dy$

Same as Exp-2

Do itself.