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# Lecture Notes - 29

BCA I year (II Sem)

Subject - Mathematics

Unit - 6

Topic - multiple Integral.

**Exp-6** Evaluate $\int_0^b\int_0^a (x^2+y^2)\,dx\,dy$

**Solution**

$$\int_0^b\int_0^a (x^2+y^2)\,dx.dy = \int_0^b\left[\int_0^a (x^2+y^2)\,dy\right]dx$$

$$= \int_0^b \left[x^2 \cdot y + \frac{y^3}{3}\right]_0^a dx$$

$$= \int_0^b \left(ax^2 + \frac{a^3}{3}\right) dx$$

$$= \left[a\frac{x^3}{3} + \frac{a^3}{3}\cdot x\right]_0^b$$

$$= \left(a\cdot\frac{b^3}{3} + \frac{a^3}{3}\cdot b\right)$$

$$= \frac{1}{3}ab(b^2+a^2) \quad \text{Ans}$$

**Exp-5** Evaluate $\int_0^3\int_1^2 xy(1+x+y)\,dx.dy.$

Do itself

Exp-8 Evaluate $\int_0^1 \int_0^x e^{y/x}\, dx\, dy$

Solution $\int_0^1 \int_0^x e^{y/x}\, dx\, dy = \int_0^1 \left[ \int_0^x e^{y/x}\, dy \right] dx$

$$= \int_0^1 \left[ x e^{y/x} \right]_0^x dx$$

$$= \int_0^1 x(e-1) \cdot dx$$

$$\Rightarrow (e-1) \int_0^1 x\, dx$$

$$\Rightarrow \int_0^1 \int_0^x e^{y/x}\, dx\, dy = (e-1) \left[ \frac{x^2}{2} \right]_0^1$$

$$= \frac{(e-1)}{2} \quad \text{Ans}$$

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Exp 9 Evaluate $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y)\, dy\, dx$

Do itself.

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**Exp-4** Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx \cdot dy}{1+x^2+y^2}$

**Solution** $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{(1+x^2)+y^2} \, dx \cdot dy$

$$\Rightarrow \int_0^1 \left[ \int_0^{\sqrt{1+x^2}} \frac{1}{(\sqrt{1+x^2})^2 + y^2} dy \right] \cdot dx$$

$$\Rightarrow \int_0^1 \left[ \frac{1}{\sqrt{1+x^2}} \tan^{-1} \frac{y}{\sqrt{1+x^2}} \right]_0^{\sqrt{1+x^2}} dx$$

$$\because \int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad \left[ \text{Where } a = \sqrt{1+x^2} \right.$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1+x^2}} \left\{ \left( \tan^{-1} \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} - \tan^{-1} 0 \right) \right\} dx$$

$$\Rightarrow \cancel{\frac{1}{\sqrt{1+x}}} \int_0^1 \frac{1}{\sqrt{1+x^2}} \left( \tan^{-1} 1 - \tan^{-1} 0 \right) dx$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1+x^2}} \left( \frac{\pi}{4} - 0 \right) dx = \frac{\pi}{4} \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$$

$$\Rightarrow \frac{\pi}{4} \log \left[ x + \sqrt{1+x^2} \right]_0^1$$

$$\Rightarrow \frac{\pi}{4} \left[ \log(1+\sqrt{2}) - \log 1 \right] = \frac{\pi}{4} \left[ \log(1+\sqrt{2}) - 0 \right]$$

$$= \frac{\pi}{4} \log(1+\sqrt{2}) \quad \text{Ans}$$